Written Exam at the Department of Economics summer 2021

Economics of the Environment and Climate Change

Final Exam

June 10, 2021

(3-hour closed book exam)

Answers only in English.

This exam question consists of 5 pages in total, including this front page.

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Written exam in the Economics of the Environment and Climate Change, Spring 2021

Question 1. Optimal environmental taxation in an economy with inequality (Indicative weight: ¾)

We consider an economy with two production sectors and two groups of workers, skilled and unskilled. The unskilled workers are employed in sector 1 producing good 1, while the skilled workers are employed in sector 2 producing another good 2. By working one hour an unskilled worker can produce one unit of good 1. The (exogenous) number of unskilled workers is n, and each unskilled worker works for ℓ hours, so the total output of good 1 is $n\ell$. Each unskilled person consumes the amount x_1 of good 1, while a skilled person consumes the amount X_1 of that good. The (exogenous) number of skilled workers is N, so the total consumption of good 1 is $nx_1 + NX_1$. Since total consumption must equal total output, we have the resource constraint

$$n\ell = nx_1 + NX_1. \qquad (1)$$

In sector 2 a skilled person can produce one unit of good 2 by working one hour. Each skilled person works for L hours, so the total output of good 2 is NL. An unskilled person consumes the amount x_2 of good 2, and a skilled person consumes the amount X_2 of that good, so by analogy to (1) we have the additional resource constraint

$$NL = nx_2 + NX_2. \qquad (2)$$

The preferences of an unskilled person are given by the utility function

$$u = \frac{x_1^{1-\eta_1}}{1-\eta_1} + \frac{x_2^{1-\eta_2}}{1-\eta_2} - \ell - aE, \qquad a > 0, \qquad \eta_1 \neq 1, \qquad \eta_2 \neq 1, \qquad (3)$$

where E is the emission of a pollutant, and the parameters η_1 and η_2 are the elasticities of the marginal utility of the two consumption goods. The presence of the term $-\ell$ in (3) reflects the disutility from work, and the term $-aE$ captures the negative welfare effect of pollution. Similarly, a skilled person has the utility function

$$U = \frac{X_1^{1-\eta_1}}{1-\eta_1} + \frac{X_2^{1-\eta_2}}{1-\eta_2} - L - bE, \qquad a > b > 0. \qquad (4)$$

The assumption $a > b$ reflects that the emissions of the pollutant are more damaging to the unskilled than to the skilled workers, say, because the unskilled live in neighbourhoods that are more exposed to pollution. Pollution is caused by the production and/or consumption of good 2, and we choose units such that the production/consumption of one unit of good 2 generates 1 unit of the pollutant. Since the total production/consumption of good 2 is $nx_2 + NX_2$, the total emissions are

$$E = nx_2 + NX_2. \qquad (5)$$

Total social welfare SW is given by the utilitarian social welfare function

$$SW = nu + NU. \tag{6}$$

For the moment, we imagine that resource allocation is controlled by a benevolent social planner who maximizes the social welfare function (6) with respect to $x_1, x_2, \ell, X_1, X_2, L$ subject to the resource constraints (1) and (2). Using (1) through (6), we can write the Lagrangian $\mathcal{L}$ for this problem as

$$\mathcal{L} = n\left[\frac{x_1^{1-\eta_1}}{1-\eta_1} + \frac{x_2^{1-\eta_2}}{1-\eta_2} - \ell - a(nx_2 + NX_2)\right] + N\left[\frac{X_1^{1-\eta_1}}{1-\eta_1} + \frac{X_2^{1-\eta_2}}{1-\eta_2} - L - b(nx_2 + NX_2)\right]$$

$$+\lambda_1(n\ell - nx_1 - NX_1) + \lambda_2(NL - nx_2 - NX_2), \tag{7}$$

where λ_1 and λ_2 are Lagrange-multipliers associated with the two resource constraints.

Question 1.1: Derive the first-order conditions for the solution to the social planner's problem.

Question 1.2: Show that the first-order conditions derived in Question 1.1 imply that the socially optimal resource allocation must satisfy the conditions

$$x_1 = X_1, \tag{8}$$

$$x_2 = X_2, \tag{9}$$

$$x_2^{-\eta_2} = X_2^{-\eta_2} = 1 + c, \qquad c \equiv an + bN. \tag{10}$$

Explain the economic intuition behind the conditions (8), (9), and (10).

We now assume that resource allocation is not determined by a social planner, but by market mechanisms influenced by taxes and subsidies. The government imposes a unit tax at the rate t_1 on consumption of good 1 and a unit tax at the rate t_2 on consumption of good 2. To compensate for the relatively low wage of unskilled workers, the government also grants a wage supplement at the rate s per hour worked by an unskilled person. Finally, the government levies a uniform lump sum tax at the rate T per person to balance its budget. Hence the condition for a balanced government budget (the government budget constraint) is:

$$\overbrace{sn\ell}^{\substack{\text{Expenditure on} \\ \text{wage supplement}}} = \overbrace{t_1(nx_1 + NX_1) + t_2(nx_2 + NX_2)}^{\text{Revenue from consumption taxes}} + \overbrace{(n+N)T}^{\substack{\text{Revenue from} \\ \text{lump sum tax}}}. \tag{11}$$

The hourly wage rate for the unskilled workers employed in sector 1 is w, whereas the skilled workers employed in sector 2 earn the hourly wage $W > w$. Since it takes one hour of work to produce one unit of output in each sector, and labour is the only production factor, the marginal costs of production in sectors 1 and 2 are w and W, respectively. The firms in the two sectors are subject to perfect competition, so producer prices are equal to marginal costs. Hence the producer prices p_1 and p_2 in sectors 1 and 2 are $p_1 = w$ and $p_2 = W$, respectively, so in equilibrium firms earn zero profits. Accounting for consumption taxes and for the government wage supplement and the lump sum tax, the household budget constraint for an unskilled worker is

$$(p_1 + t_1)x_1 + (p_2 + t_2)x_2 = (w + s)\ell - T, \tag{12}$$

whereas the budget constraint for a skilled worker is

$$(p_1 + t_1)X_1 + (p_2 + t_2)X_2 = WL - T, \tag{13}$$

since skilled workers do not receive any wage supplement from the government.

Question 1.3: An unskilled worker maximizes her utility function (3) with respect to x_1, x_2 and ℓ subject to her budget constraint (12), taking prices, taxes, the net wage $(w + s)$ and the total emission level E as given. Set up the Lagrangian corresponding to the unskilled worker's maximization problem (denote the Lagrange multiplier associated with the budget constraint by λ^u) and show that the first-order conditions for the solution to the unskilled worker's problem imply that

$$x_1 = \left(\frac{p_1+t_1}{w+s}\right)^{-\frac{1}{\eta_1}}, \tag{14}$$

$$x_2 = \left(\frac{p_2+t_2}{w+s}\right)^{-\frac{1}{\eta_2}}. \tag{15}$$

By following a procedure similar to the one in Question 1.3, one can show that a skilled person's utility-maximizing consumption levels are

$$X_1 = \left(\frac{p_1+t_1}{W}\right)^{-\frac{1}{\eta_1}}, \tag{16}$$

$$X_2 = \left(\frac{p_2+t_2}{W}\right)^{-\frac{1}{\eta_2}}. \tag{17}$$

Question 1.4: Use eqs. (14) through (17) to derive a formula for the wage supplement s and a formula for the consumption tax rate t_2 which will ensure that the market economy generates the socially optimal allocation satisfying eqs. (8), (9), and (10). (Hint: Remember that because of perfect competition, $p_2 = W$).

Question 1.5: Give an economic interpretation of (explain the intuition for) your formula for the optimal environmental tax rate t_2 derived in Question 1.4. (Hint: Utility maximization can be shown to imply that $\lambda^s = 1/W$, where λ^s is a skilled worker's marginal utility of income, i.e., the Lagrange multiplier associated with her budget constraint. Moreover, under the optimal policy we have $\lambda^u = \lambda^s$, where λ^u is an unskilled worker's marginal utility of income. Given these insights, what is an unskilled worker's marginal willingness to pay (MWTP) for a cut in pollution, measured in monetary units? And what is a skilled worker's MWTP?).

Question 1.6: From an environmental viewpoint, or from a fiscal viewpoint, is there any reason why the government should choose to impose a tax t_1 on the consumption of good 1? Briefly motivate your answer.

Question 1.7: Let $\overline{N} \equiv n + N$ denote the total population, assumed to be constant, and let α denote the share of unskilled workers in the total population so that $n = \alpha\overline{N}$ and $N = (1 - \alpha)\overline{N}$. How does an increase in α affect the optimal environmental tax rate t_2? Explain. How is an increase in α likely to affect the optimal size of the wage supplement s? Explain (in verbal terms, you are not asked to undertake a mathematical analysis).

Question 1.8: Now suppose the government cannot implement a wage supplement, perhaps because it cannot observe the number of work hours of unskilled workers. Suppose further that the government does not have other policy instruments that can ensure an equalization of income distribution. Discuss briefly the considerations that are now relevant for the government's choice of the environmental tax rate t_2. Would the MWTP of the two groups of workers be given equal weight in the government's decision on the tax rate? And would the MWTP of the two groups of workers be the same? Explain.

Question 2. Trade and the environment (Indicative weight: ¼)

Discuss the relationship between international trade and the quality of the environment. (*Note: This question may be answered without any use of math and/or graphical analysis. However, you are welcome to use math or diagrams to the extent that you find it convenient*).